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# A novel method for human Astigmatism formulation and measurement

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## ABSTRACT

Astigmatism's defect has not yet been formulated in Physics and medical physics books theoretically. In this paper, we are trying to formulate the astigmatism's defect. The results of formulation show that "the degree of astigmatism is negative reverse the distance that vision's condition of astigmatism's chart lines change conditions in dimness angle from dimness condition at infinite distance or the farthest vision's distance to clarity condition at that angle". Secondly, the degree of astigmatism which has been theoretically formulated is measured with a lens with a special focus. To read the degree of astigmatism faster, we have tried to let the machine show the degrees automatically.

**Keywords**: Astigmatism, Novel Method, Formulation, Measuremen

## INTRODUCTION

Astigmatism's defect is one of the eye defects which have not been formulated in reference books of physics and medical physics (Hollins,2001; Sanchez et al,2003; Gills, 2003; Montano, 2002; Michael, 2012; Brown et al,1999; Barnett, 2012; Crundell et al, 2003). This defect had been measured with costly and developed machines according to the reflection of laser's ray from eye's surface, lens gradient-index, Hartmann sensor, infrared LEDs, Optical surface reconstruction technique, Video Keratography System and laser beam technique (Gekeler et al,1996; Seideman et al,1999; Díaz et al,2012; Ventura et al,1998; Wei et al,2009; Liliane et al,2009; Espinosa et al, 2010; Mas et al, 2009; Xu et al,2009 Mierdel et al,2001; Chang et al,2011). In this paper, first would like to formulate the astigmatism's defect by presenting a novel method based on known equations of optical physics and second to experiment this formulation.

## THEORY OF ASTIGMATISM'S FORMULATION

The astigmatism's defect is due to the fact that the sphere convergence of the eye is not equal in different angles in relation to an axis which is vertical to the main axis of eye's lenses. If was considered this axis parallel to the horizon, the focus of eye's lenses in each angle in relation to this axis will be $f_\Theta$ and the convergence of eye's sphere in this angle will be $D_\Theta = 1/f_\Theta$. The focus for the zero angle is shown

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with $f_{\parallel}$(or the convergence $D_{\parallel} = 1/f_{\parallel}$) as parallel focus and the 90 angle will be shown with $f_{\perp}$(or the convergence $D_{\perp} = 1/f_{\perp}$). If one or some of the lines of astigmatism's chart in a special angle are not seen dim or unclear, the convergence of the eyes in that angle is different from other angles. The difference between the convergence of dim lines in a special angle ($D_{\Theta}$) and clear lines which have equal convergence in other angles with a convex lens which is situated on the main axis of eye's lens and installing and moving a astigmatism chart which is used for astigmatism formulation. There are many model for human eye's (David,2006;Lotmar,1971).In this paper was used a simple eye model and was put a lens with a special focus (f) at the beginning of a rail on which the astigmatism's chart has been installed according to figure1. If the chart was seen at behind, the distance from the image to the lens (q) depends on the distance of the chart to the lens (p). The virtual image will vary from zero to the infinite if we change the distance of the chart from zero to the lens. This technique is applied due to the fact that the movement from zero to the infinite does not exist in the laboratory and we provide it by one meter moving the chart.

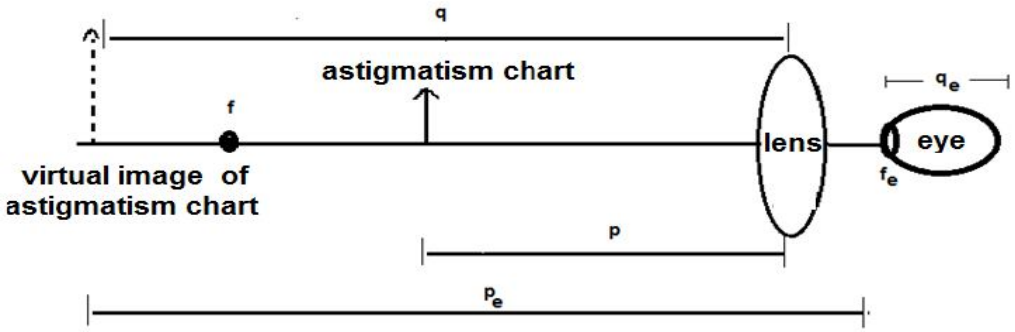


**Figure 1:** The machine and the similar picture

According to the figure1 and the lenses relationship (equation1), the following equations can be written.

$$1/f=1 / p +1/q \qquad (1)$$

The f is the focus of the used convex lens, the p is the distance of astigmatism's chart to the lens and the q is the distance of the astigmatism's chart to the eye. The virtual image of the astigmatism's chart provided by this lens is the same as a virtual object for eye. Therefore, the distance of this virtual object to eye ($p_e$) is equal to $p_e$=-q. therefore the equation 1 is changed as equation 2.

$$1/p_e=1/p- 1/f \qquad (2)$$

The equation of eye's convergence with lens focus and distance of astigmatism's chart to the lens and the diameter of eye's sphere $q_e$ that is shown by equation 3 was changed by using equations 1 and 2 as equation 4.

$$1/f_e=1 / p_e +1/q_e \qquad (3)$$

$$1/f_e=1/p- 1/f +1/q_e \quad , \quad De=1/f_e$$

$$D_e=1/p- 1/f +1/q_e \qquad (4)$$

It can show the equation 4 for the eye's convergence in each angle $D_e = D_\Theta$ as following:

$$D_\Theta = 1/p_\Theta - 1/f + 1/q_e \qquad (5)$$

Where $p_\Theta = p$ is the distance of astigmatism's chart to the focus in each angle.

It was assumed that the astigmatic eye does not see the chart's lines in Θ angle with the difference ΔΘ clearly but sees the rest of lines like zero angle or any angle except Θ with the convergence $D_0 = D_\parallel$ and the distance $p_= p_0$clearly. The difference between these two divergences according to equations 4 and 5 will be as the equations 6:

$$\Delta D = D_\Theta - D_0 = (1/p_\Theta - 1/f + 1/q_e) - (1/p_0 - 1/f + 1/q_e) = (1/p_{e\Theta} + 1/q_e) - (1/p_{e0} + 1/q_e)$$

$$\Delta D = 1/p_\Theta - 1/p_0 = 1/p_{e\Theta} - 1/p_{e0} \qquad (6)$$

If the chart is put in the focus of lens $p_\Theta = f$ (meaning in the infinite relative to the eye $p_{e\Theta} = \infty$) at first, the lines which has been put in Θ angle of astigmatism chart will not be clear. But if the chart is put in the distance x from the lens $p_0 = x$ (meaning at the distance $p_{e0} = y$ relative to the eye), the lines which have been put in Θ angle of astigmatism chart will not be clear. Therefore, the equation 6 will be changed to the following:

$$\Delta D = 1/f - 1/x \qquad (7) \quad , \quad \Delta D = 1/\infty - 1/y \quad , \quad \Delta D = -1/y \qquad (8)$$

According to the equation 7, the difference between the convergence of the lines which are seen clearly and the ones which are seen unclearly can be obtained in two ways. The first way is to put the chart in the infinite distance without using lens and mark the lines which are not clear at Θ angle. Next, the chart is transferred from the infinite to the y distance from the eye to make the marked unclear lines in the infinite distance that is seen clear. In this way, the divergence difference which is equal to the degree of cylindrical lens which must be used to fix the astigmatism's defect is equal to the negative reverse of the y distance according to the equation 8( $\Delta D = -1/y$).The second way is to put the chart in the focus of lens with a lens with the focus f and to look at the astigmatism chart from behind. The lines which are not clear at Θ angle are marked. Then, we transfer the chart from the focus to the x distance from the lens to make the marked unclear lines be seen clearly. In this way, the divergence difference which is equal to degree of the cylindrical lens which must be used to fix astigmatism's defect is equal to the following according to the equation 7 ($\Delta D = 1/f - 1/x$). In the end, according to the equation 8, the following principle is used to determine the vision degree of astigmatism's defect:

*"the degree of astigmatism is negative reverse the distance that vision's condition of astigmatism's chart lines change conditions in dimness angle from dimness condition at infinite distance or the farthest vision's distance to clarity condition at that angle" or ΔD= -1/y .*

The astigmatism's defect is due to the fact that the divergences at different angles are not equal and have equal to ΔD differences. If an eye is fine, the ΔD will be zero. With the equation 8 and putting a cylindrical lens with a convergence obtained from the equation 8, this defect will be fixed. The astigmatism's degree of eye is the degree of the lines which are not clear at the infinite.

## MEASURING THE ASTIGMATISM'S DEFECT

According to the figure 1 and equation 7, a machine for measuring astigmatism can be introduced. This machine consists of the following parts: a) a lens b) an astigmatism chart c) an engine d) a lubricate rail e) electronic of the machine and f) a holder as figure 2.

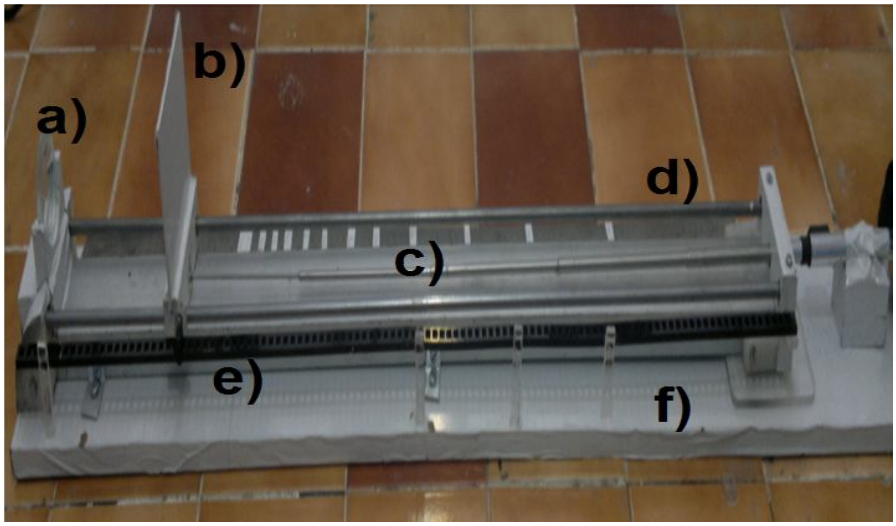


**Figure 2:** the device for astigmatism measurement a) a lens b) an astigmatism chart c) an engine d) a lubricate rail e) electronic of the machine and f) a holder

The astigmatism chart is a half-circle on which some lines with equal length at an equal angular distance have been drawn (Figure 2b).A convergent lens with the focus 105 cm has been installed on the rail. This lens, if the chart is in the focal distance, shows a virtual image of it between zero and the infinite (Figure 2a). The rail consists of two metal bars which are installed on some holders as shown in the figure 2d.A car antenna installed on the astigmatism chart have used to move and change direction of the chart (Figure 2c).The holder consists of an aluminum can with the dimensions of $4\times8\times115$ $cm^3$ and a wooden sheet with the dimension of $1\times8\times115$ $cm^3$ (Figure 2f).The electronic of the machine consists of the machine's software and hardware. The hardware is a range finder circuit which works with two infrared sensors, one receiver and one transmitter. The transmitter is installed on the body of the machine and the receiver on the chart (figures 2e and 3). One IC, with the capability of planning information receives the distance from the range finder and shows on the screen. Also, the IC circuit has the capability to exchange the received distances from astigmatism, hyperopia and myopia in the relevant equations and show the degree of astigmatism, hyperopia and myopia on the screen.

**Figure 3.** Electronic of Machine Consisting of Software and Hardware

To measure the degree of eyes astigmatism, a lens with the focus of 105 cm on a horizontal lubricate with a length more than 105cm was installed. The astigmatism chart was moved from the focus to the point that vision's condition of astigmatism's chart lines in dimness angle change condition from dimness to clarity at that angle. The chart is movable by an engine. By using the equation 7, astigmatism's defect is measurable.

## RESULTS AND DISCUSSION

The equation degree of astigmatic eye can be measured by putting the focal distance f=105 cm in the equation 7.

$$\Delta D= 1/1.05-1/x \qquad (9)$$

The absolute value of degree is drawn as a function of chart's distance to the lens in figure 3. the sign of degree of astigmatism is negative amounts of figure 3.

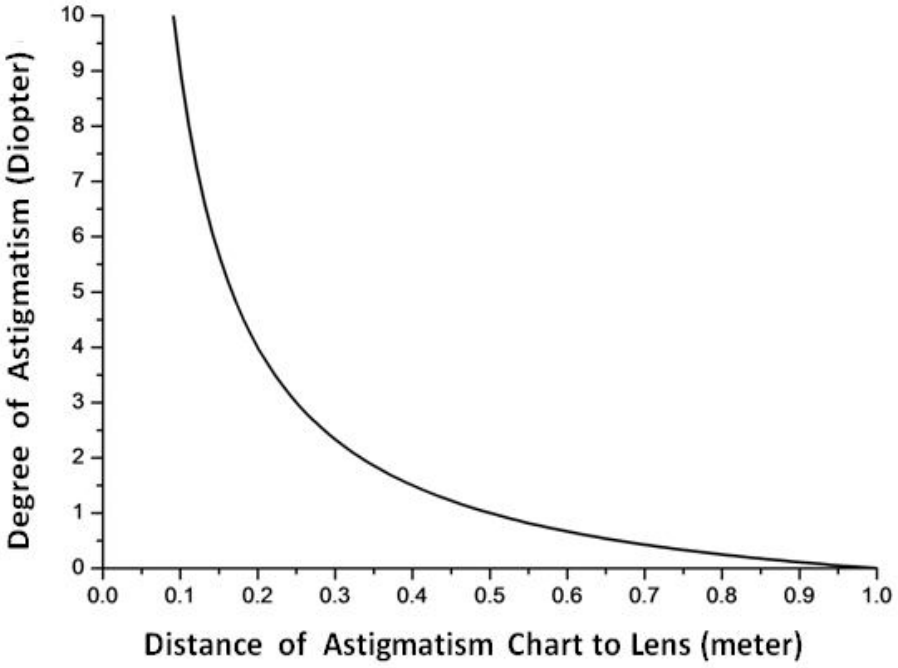


**Figure 3.** Degree of Astigmatic vs Chart Distance to the Lens with 105cm Focus

The minimum and maximum level for human astigmatism degree was reported as zero and -2.5 diopter respectively (Jafarzadehpur et al, 2012).The relevant data to this formulation is depicted between zero and -10 diopter in table 1.

**Table 1:** The Data of the Eye Astigmatic versus the Chart Distance to Lens.

| **Astigmatic degree (diopter)** | **chart distance to lens (cm)** | **Astigmatic degree (diopter)** | **chart distance to lens (cm)** |
|---|---|---|---|
| 0 | 105 | -3.75 | 21 |
| -0.25 | 83 | -4 | 20 |
| -0.5 | 70 | -4.25 | 19 |
| -0.75 | 60 | -4.5 | 18 |
| -1 | 51 | -5 | 16.5 |
| -1.25 | 45 | -5.5 | 15 |
| -1.5 | 41 | -6 | 14 |
| -1.75 | 37 | -6.5 | 13.5 |
| -2 | 34 | -7 | 12.5 |
| -2.25 | 31 | -7.5 | 11.5 |
| -2.5 | 29 | -8 | 11 |
| -2.75 | 27 | -8.5 | 10.5 |
| -3 | 25 | -9 | 10 |
| -3.25 | 24 | -9.5 | 9.5 |
| -3.5 | 22.5 | -10 | 9 |

## THE ANALYSIS OF PROBABLE ERRORS

If we cannot obtain the focus of the lens put in front of the eyes, some errors occur in calculation of degree of astigmatic eye. We suppose that there is a 5cm error in measuring the lens focal distance put in front of the eyes. Therefore, we will show the probable error in calculation of degree of astigmatic eye in table 2.

**Table 2 .** The probable error in calculation degree of astigmatic eye due to the error in calculation of lens focal distance .

| degree (diopter) | Error in measuring the focus of lens in a 100cm focus | | | Dingle of degree correctness Cm |
|---|---|---|---|---|
| | F=95 cm | F=100 cm | F=105 cm | |
| | X cm The chart distance to lens | X cm The chart distance to lens | X cm The chart distance to lens | |
| 0 | 95 | 100 | 105 | 95-105 |
| 0.25 | 76.76768 | 80 | 83.16831 | 77-83 |
| 0.5 | 64.40678 | 66.66666 | 68.85246 | 65-69 |
| 0.75 | 55.47445 | 57.14286 | 58.74126 | 56-59 |
| 1 | 48.71795 | 50 | 51.21951 | 49-51 |
| 1.25 | 43.42857 | 44.44444 | 45.4054 | 43-45 |
| 1.5 | 39.17526 | 40 | 40.7767 | 39-41 |
| 1.75 | 35.68075 | 36.36364 | 37.00441 | 36-37 |
| 2 | 32.75862 | 33.33333 | 33.87097 | 33-34 |
| 2.25 | 30.27888 | 30.76923 | 31.22676 | 30-31 |
| 2.5 | 28.14815 | 28.57143 | 28.96552 | 28-29 |
| 2.75 | 26.29758 | 26.66667 | 27.00965 | 26-27 |
| 3 | 24.67533 | 25 | 25.3012 | 25-25.5 |
| 3.25 | 23.24159 | 23.52941 | 23.79603 | 23-24 |
| 3.5 | 21.96532 | 22.22222 | 22.45989 | 22-22.5 |
| 3.75 | 20.82192 | 21.05263 | 21.26582 | 21-21.25 |
| 4 | 19.79167 | 20 | 20.19231 | 20-21 |
| 4.25 | 18.85856 | 19.04762 | 19.22197 | 19-19.25 |
| 4.5 | 18.00948 | 18.18182 | 18.34061 | 18-18.5 |
| 4.75 | 17.23356 | 17.3913 | 17.53653 | 17-17.5 |
| 5 | 16.52174 | 16.66667 | 16.8 | 16.5-17 |
| 5.25 | 15.86639 | 16 | 16.12284 | 16-16.5 |
| 5.5 | 15.26104 | 15.38461 | 15.49815 | 15.25-15.5 |
| 5.75 | 14.70019 | 14.81481 | 14.92007 | 14.75-20 |
| 6 | 14.1791 | 14.28571 | 14.38356 | 14-14.5 |
| 6.25 | 13.69369 | 13.7931 | 13.8843 | 13.75-14 |
| 6.5 | 13.24042 | 13.33333 | 13.41853 | 13.25-13.5 |
| 6.75 | 12.81619 | 12.90323 | 12.983 | 12.8-13 |

| | | | | |
|---|---|---|---|---|
| 7 | 12.4183 | 12.5 | 12.57485 | 12.5-12.75 |
| 7.25 | 12.04437 | 12.12121 | 12.19158 | 12-12.25 |
| 7.5 | 11.69231 | 11.76471 | 11.83099 | 11.75-12 |
| 7.75 | 11.36024 | 11.42857 | 11.49111 | 11.25-11.5 |
| 8 | 11.04651 | 11.11111 | 11.17021 | 11-11.25 |
| 8.25 | 10.74965 | 10.81081 | 10.86675 | 10.75-11 |
| 8.5 | 10.46832 | 10.52632 | 10.57934 | 10.4-10.75 |
| 8.75 | 10.20134 | 10.25641 | 10.30675 | 10-10.3 |
| 9 | 9.94764 | 10 | 10.04785 | 9.9-10 |
| 9.25 | 9.70626 | 9.7561 | 9.80163 | 9.7-9.8 |
| 9.5 | 9.47631 | 9.52381 | 9.5672 | 9.5-9.6 |
| 9.75 | 9.257 | 9.30232 | 9.34371 | 9.25-9.4 |
| 10 | 9.04762 | 9.09091 | 9.13044 | 9-9.2 |

As shown in table 2, it is possible to measure the astigmatic degree even if the lens focal distance is not exact. For example, if the chart is put in the 43 to 45 cm distance and the clarity condition of the line is reversed, the astigmatic degree will be 2.25 diopter. After determining the machine's errors by referring to optometrist and eye doctor's centers of Kerman city, the measurement of astigmatism of some guests was investigated. The results are shown in figure 4. In this figure, the amounts measured by the machine are compared with the amount measured by optometrists and eye doctors.

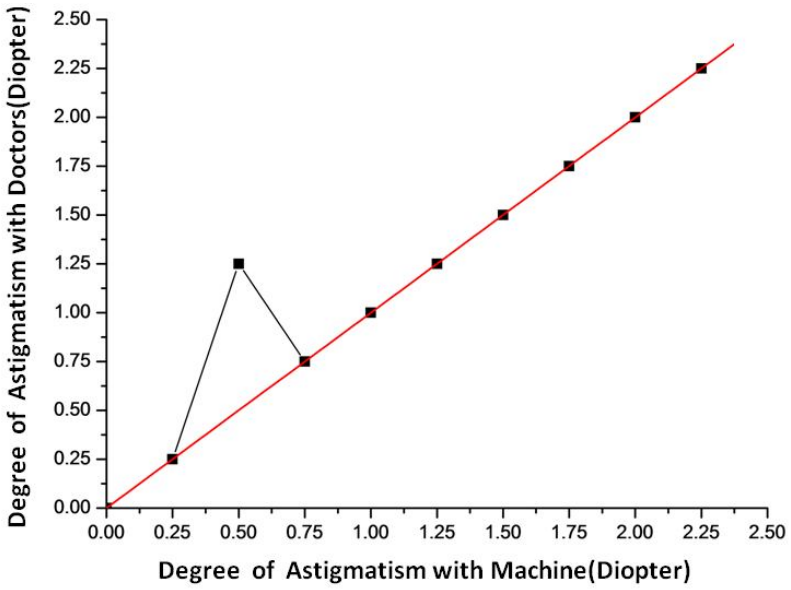


**Figure 4.** the degree Measured by the Machine , Doctors and Optometrists in Kerman City.

The Comparison of the degree Measured by the Machine , Doctors and Optometrists in Kerman City shows the accuracy and correctness of the introduced formulation and the produced machine. The figure 5 also shows that the measured amounts by the machine (bullet points) with the extracted amounts by the introduced formulation (dashed lines).

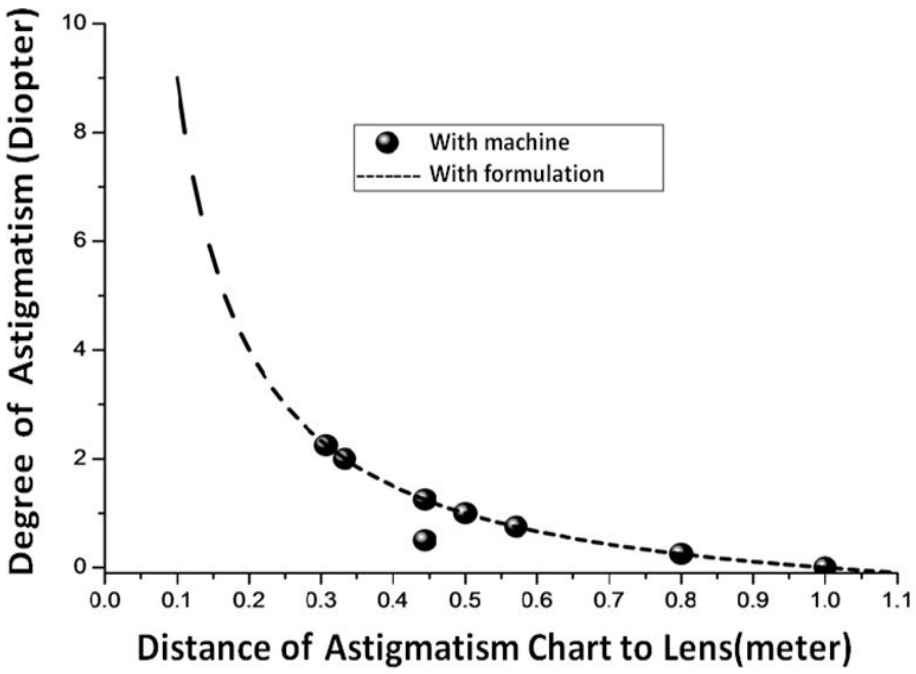


**Figure 5.** The Comparison between the results obtained by the machine and the formulation

The case is when the eyes of astigmatic person have the hyperopia's defect, too. Therefore, the presented formulation with the presented machine has the capability of measuring the astigmatic eye perfectly, so by industrial production, it can equip the eye doctor's centers with this machine with low prices to measure the astigmatic degree. By means of an infrared range finder circuit which has been combined with a programmable IC, it can show the astigmatism, the far-sighted astigmatism, and near-sight astigmatism on the screen. To do this, it just needs to move the astigmatism chart from the focus to the lens. The first distance where most of the astigmatism lines are seen is shown by $x_N$, the second distance where the ambiguous lines of the astigmatism line are clear is shown by $x_A$, and the third distance where all the lines are dim is shown by $x_A$. By putting these three in the relevant equations, the degree of the astigmatism, the far-sighted astigmatism, and near-sighted astigmatism are shown.

## Conclusion

At first, the astigmatic eye's defect has been analyzed in two stages theoretically. In the first stage, the equation of the astigmatic degree is obtained. The degree of astigmatism is negative reverse the distance that vision's condition of astigmatism's chart lines change conditions in dimness angle from dimness condition at infinite distance or the farthest vision's distance to clarity condition at that angle or $\Delta D = -1/y$. At the second stage, the astigmatism's defect has been scaled by means of a lens with a 105cm focus. The relation between astigmatism degree $\Delta D$ and the chart distance to the lens x is $\Delta D = 1/x - 1/1.05$.At last, by combining an infrared range finder and a programmable IC, the astigmatic degree is measured automatically, and by industrial production we can equip the eye doctor's centers, schools and research centers with this to measure the degree of astigmatism cheaply.

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