Simulation shrinkage and stress generated during convective drying of carrot slices

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Abstract

The moist sample shrinks during drying and the non-uniformity shrinkage involves stresses in the body. The aim of the present work is simulating deformations and drying-induced stresses of carrot slice during a convective drying. The effect of shrinkage on the heat and mass transfer profiles were considered. Non-constant physical and thermal properties were also incorporated in the model and the Experimental conditions were as follows: a drying temperature of 333 K, relative humidity of 11% and air velocity of 1.2 m/s.

Key word: Drying-induced stresses, shrinkage, carrot

INTRODUCTION

One of the most important physical changes that occur in the food industry during drying is the reduction of its external volume. Loss of water and heating cause stresses in the cellular structure of the food leading to change in shape and decrease in dimension so study of heat and mass transfer is needed in the simulation of shrinkage and drying-induced stresses. Forced convection by hot and dry air is the most widespread industrial technique to execute food drying. Drying is a complex process involving simultaneous coupled transient heat and mass transfer. It is a process whereby the moisture is vaporized and swept away from the surface (Barati and Esfahani, 2011). Carrot is one of the most common used vegetables for human nutrition due to the high vitamin and fiber content. Carrots have a variety of health effects; they are rich in vitamins, sugar, starch, potassium, calcium, phosphorus, iron and other nutrients and inorganic salts and five kinds of essential amino acids (aboltins *etal*., 2011). Fresh carrot has moisture content close to 80–90%(Mihoubi,D *etal*., 2009) so during drying carrot samples lost a lot of water that it caused shrinkage phenomenon be most important in the carrot drying. The strain and stresses are caused when temperature and moisture gradient are generated in materials whose volume changes with heating and moisture removal. In such materials, failure and irregular deformation may be generated which affect considerably the quality of the products after drying. The general constitutive equation for strain and stresses are often analyzed with the strain behavior given by a function of moisture (Hasalani and Itaya, 2007). Agricultural products and specially root vegetables undergo several physical and

structural modifications during the drying process. Shrinkage of root vegetables during drying is important not only from the viewpoint of material end-use but also for simulation problem. Surface cracking is another phenomenon that may occur during drying. This happens when shrinkage is not uniform during the drying process leading to the formation of unbalanced stresses and failure of the material (Mayor and Sereno, 2004). A model of simultaneous heat and moisture transfer in a cylindrical sample was coupled with the virtual work principle applicable to a body undergoing shrinkage deformation in two dimensions. Non-constant physical and thermal properties were also incorporated in the model. Governing equations and boundary conditions were solved numerically using Galerkin's finite element method (Yang *etal.,* 2001). (Aversa and etal, 2007) presented a theoretical model describing the transport phenomena involved in food drying. Trey reported that comparison between the theoretical predictions and a set of experimental results has shown a remarkable agreement especially during the first 2 h when experimental data and theoretical predictions overlapped and relative errors never exceeded 5%. Later on, a slight deviation can be observed, particularly for the drying test performed with higher air temperature and smaller food thickness, i.e. when a significant volume reduction (shrinkage) was possibly achieved. The observed deviation can be, therefore, ascribed to the lack of accounting for shrinkage effects in the model. The most of the models were done in the one or two dimensions and shrinkage was neglected. The objective of this study was to develop a drying model that takes into account three-dimension shrinkage formation and consider the effect of shrinkage on the heat and mass transfer profiles and obtained the stress profile during drying. Using cylindrically formed carrot samples, experimental drying data were obtained at a drying temperature 333 K a relative humidity of 11% and air velocity 1.2 m/s. Predicted values were compared to experimental results and good agreement were between them.

MATERIAL AND METHODS

A cylindrical carrot slice with diameter of 1cm and thickness of 4mm was considered. The initial temperature and moisture content of product were 20°C and 7.8(gr gr^{-1}), respectively. Air properties are assumed constant and equal to 60°c and 11% for temperature and moisture content, respectively. The validation of the model is made with a set of numerical and experimental results reported in the literature for carrot sliced in slab form (Rostami,R etal,. 2013)

Mathematical Model

Temperature and moisture gradient between the food and air are caused the heat and mass transfer. Hence, a mathematical model was developed to predict the moisture content, temperature, shrinkage and principle stress profile of carrot undergoing convective dryer.
To simplify the model, the following assumptions are made:
- Carrot was assumed to be continuous, homogeneous and isotropic material
- Initial distribution of moisture and temperature were considered to be uniform
- Mass transfer in the product by diffusion
- Moisture evaporation occurred only at the external surface of carrot
- Stress- strain relation obeyed Hook's constitutive equation of elastic behavior

Heat Transfer
The heat transfer through a food is induced because of the temperature gradient between the food and air (Mohan and Talukdar, 2010).
The energy balance in the solid, based on Fourier's law, leads to

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) \tag{1}$$

Initial condition: $T = T_0$ t=0

Boundary condation:

$$k \frac{\partial T}{\partial n} = h_T \, (T_a\text{-}T_{surf}) - m'.L_v \qquad\qquad (2)$$

It is important to note that the term on the left side of Eq. (2) refers to heat conducted from the outer surface to the inside of the body, the first term on the right side is heat penetrating from the environment to the food by means of convection, and the second term on the right side denotes heat evaporation.

Moisture Transfer

A three-dimensional Fickian diffusion model of moisture transfer is applied to simulate the time evolution of the local moisture content, spatial distribution, in the carrot during drying (Barati and Esfahani, 2013).

$$\frac{\partial M}{\partial t} = \nabla.(D \nabla M) \qquad\qquad (3)$$

Initial condition: $M = M_0$ $t = 0$

When the moisture migration from the fresh carrot slice was partly affected by the humidity of the drying air, it was necessary to consider the boundary condition that kept a rate balance between evaporation and the internal moisture diffusion. Taking external mass transfer into account, the rate of moisture evaporation from the surface of carrot can be represented as follows:

$$-D_{eff}\, \rho \, \frac{\partial M}{\partial n} = h_m\,(M_i - M_{\infty}) \qquad\qquad (4)$$

Where M_i is the moisture concentration at the carrot surface, M_{∞} is the equilibrium moisture concentration with the drying air

Equations of stress field

Total displacements {dU} at any point in the sample during a finite time increment are axisymmetric, as expressed by Eq. (5) (Yang *etal.,* 2001).

$$\{dU\} = \begin{Bmatrix} du \\ dv \\ dw \end{Bmatrix} \qquad\qquad (5)$$

Local Strain

Local total strains $\{d\varepsilon\}$ are function of changes in mechanical strains $\{d\varepsilon_s\}$ (a constrained deformation due to elasticity) and free shrinkage strains $\{d\,\varepsilon_0\}$ (the sum of a free deformation due to moisture loss), as expressed by Eq. (6). (Hasalani and Itaya, 2007).

$$\{d\varepsilon\} = \{d\varepsilon_s\} + \{d\varepsilon_0\} \qquad\qquad (6)$$

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Strain–Displacement Relation

Total strain $\{d\varepsilon\}$ is a function of total displacements $\{dU\}$, as expressed by Eq. (7).

$$\{d\varepsilon\} = [B]\{dU\} \tag{7}$$

Stress–Strain Relation

Stresses are function of changes in mechanical strains $\{d\varepsilon_s\}$ as expressed by Eq. (8).

$$\{d\sigma\}=[D]\{d\varepsilon_s\} \tag{8}$$

Relationship between Shrinkage Strain Increment and Directional Shrinkage Coefficient a free shrinkage strain increment $\{d\varepsilon_0\}$ related to the shrinkage coefficient due to moisture loss, as expressed by Eq.9.

$$\{d\varepsilon_0\} = \begin{Bmatrix} d\varepsilon_{xx} \\ d\varepsilon_{yy} \\ d\varepsilon_{zz} \end{Bmatrix} = \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} \tag{9}$$

Sx, Sy, and Sz are free shrinkage coefficients in x, y and z direction for a carrots sample.

Virtual Work Principle

In the stress analysis, an equilibrium equation and mechanical boundary condition are necessary in addition to the above equations.

The virtual work principle is used to obtain the equilibrium equation. The following variation was obtained when this principle was applied assuming that zero body and surface forces are applied to food, the following equilibrium equation is obtained:

$$\int_v \delta\{d\varepsilon^s\}^T \{d\sigma\}dV = 0 \tag{10}$$

$$\int [B]^T[D][B]dv\{dn\} = \int [B]^T[D]\{d\varepsilon^d\}dv \tag{11}$$

When Eq.(11) was solved numerically, a set of equation were obtained. These equations were solved to obtain nodal displacements. After $\{dn\}$ was obtained the total strain was calculated and the free strain was estimated from the observed and with Eq.6 mechanical strain was found. Finally, a stress field was obtained. (tsuka *etal.*,1991; Niamnuy *etal.*,2008; Curcio and Aversa, 2009).

Parameter Used in the Model

Transport parameters and various physical properties were required during the solution of the governing equations so they were taken from other literature.

Heat transfer coefficient (h_t)
Heat transfer coefficient (h_t) was used in the boundary condition of heat transfer it is calculated on the basis of well-known semi-empirical correlation expressing the dependence of Nusselt number upon Reynolds and Prandtl numbers,

$$Nu = h.L / K_{air} = 0.664 * Re^{1/2} * Pr^{1/3}$$

Where Nu, Re, Pr represent Nusselt number, Reynolds number, Prandtl number, respectively

Mass transfer coefficient (h_m)
 Hence heat and mass transfer have relations with each other. Therefore, obtaining mass transfer coefficient is effortless after having convective heat transfer coefficient, and applying the similarity of heat and mass transfer as the following

$$h/h_m = \rho_{air} c_{air} Le^{.66}$$

Where Le is Lewis number.

Thermal conductivity
The Thermal conductivity (k) of carrot was determined by(lopez etal.,2004)

$$K = 0.49 - 0.443 \exp(-.206 * M)$$

Specific heat (c_p)
The Specific heat (c_p) of carrot was determined by(Mihoubi,D *etal*., 2009)

$$c = 837 + 4180 M/(M+1)$$

Effective moisture diffusion coefficient (D_{eff})

The effective moisture diffusion (D_{eff}) followed the form of correlation adopted by (Białobrzewski,I *etal*., 2008) and is represented as

$$D_{eff} = 2.779 * 10^{-4} * \exp(-.97 - (3459.8/T) + .059 * M)$$

Water activity

Water activity was determined by GAB model.

$$w = \frac{w_m c\, k\, a_w}{(1 - k a_w)[1 + (c-1) k\, a_w]}$$

The constants were determined through the following Arrhenius form:

$$W_m = w_{m0} \exp\left(\frac{w_{ma}}{T}\right)$$

$$C = C_0 \exp\left(\frac{c_{0a}}{T}\right)$$

$$K = K_0 \exp\left(\frac{k_{0a}}{T}\right)$$

The coefficients w_{m0}, c_0, k_0, w_{m1}, c_{01} and k_{01} were equal to 0.014, 1.05×10^{-6}, 1.18, 695.67, 6313.74 and -60.60, respectively (Mihoubi,D *etal*., 2009).

Volumetric moisture shrinkage coefficient (S_v)

Volume of carrot was determined using a liquid pycnometer with n-heptan as the working liquid. $S_{v} = v_t / v_0$ is the volumetric moisture shrinkage that was determined experimentally and correlated with the moisture content through the following equation (Rostami, R *etal*., 2013).

$$v/v_0 = 0.0103\ M^2 + 0.035M + 0.098$$

RESULT AND DISCUSSION

The simulated results of temperature, moisture content and volume shrinkage were compared with the experimental results. The principal stress and moisture gradient profile of carrots slice during the convective drying are obtained. (Fig.1) shows the evolution of the average moisture content of carrot obtained for drying kinetic simulated at 60°C with a relative humidity of 11% and an air velocity of 1.2m/s. During drying moisture in carrot migrated from the inside to the surface of carrot that the rate of moisture movement was controlled by moisture diffusion rate through the product. The rapid decrease of moisture content in the early drying period is attributed to the high moisture gradient. It was found that the trends of the moisture content prediction were in very good agreement with the experimental data, particularly in the period, when the shrinkage assumption was used. This phenomenon was observed by Aversa etal,(2007).

(Fig.2) shows the variation of temperature with time. The temperature inside the wet product increases during drying. Rapid rise of temperature observed at the first of the drying period, the temperature then slowly increase because the latent heat of evaporation prevent rapid temperature rise. It was found that the trends of the temperature prediction were in very good agreement with the experimental data.
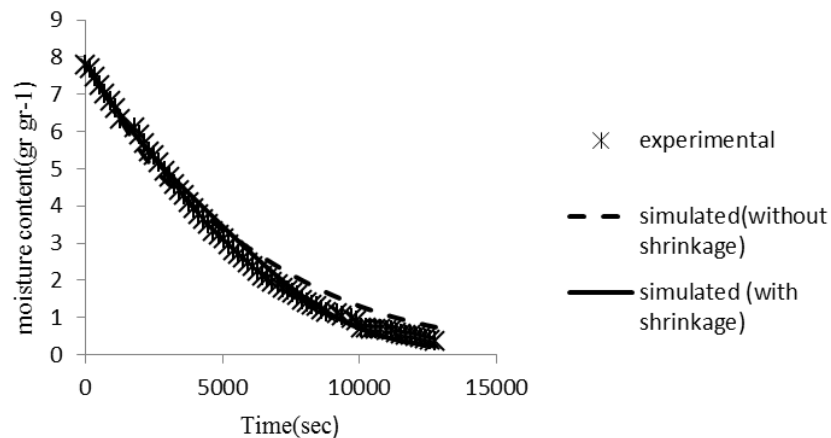

Fig.1. comparison between simulated and experimental moisture content of carrot at drying temperature of 60°c
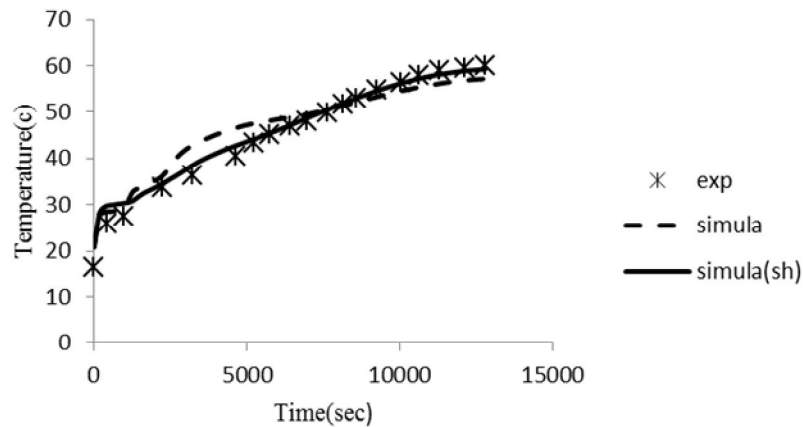
Fig.2. simulated and experimental up-layer temperature of carrot at drying temperature of 60°c

(Fig.3) shows the comparison between the simulated and experimental volume changes (V / V0) of cylindrical carrot samples during the convective drying. When water is removed from the samples during drying, a force unbalanced is produced between the inner and other parts of the sample and external force leads to shrinkage.
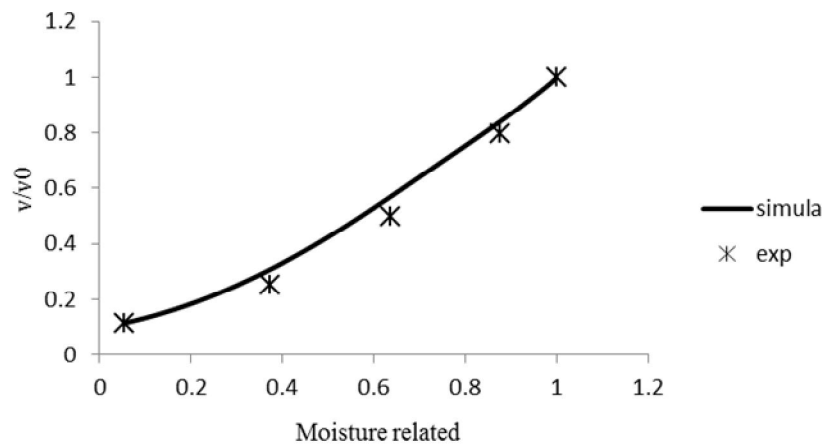

Fig.3. simulated and experimental volume shrinkage as a function of moisture related at drying temperature of 60°c

(Fig.4) present the simulated maximum principal tensile stress versus moisture content of carrot slices during drying. It can be observed from fig.4. that the maximum principle tensile stress at the first time was equal to zero because the initial moisture content was uniform then increase to the highest value after that its decrease .This is because of the changes of the maximum moisture gradient of carrot during drying (fig.5) this phenomenon was observed by Niamnuy *etal*.,(2008)
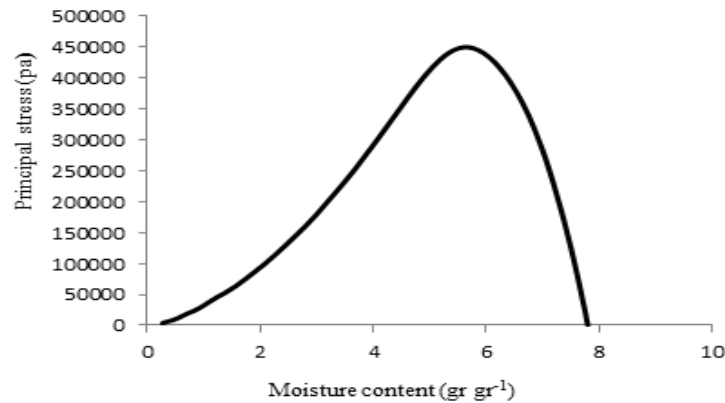
Fig.4. relationship between maximum principal stress and moisture content of carrot at drying temperature of 60ºc.
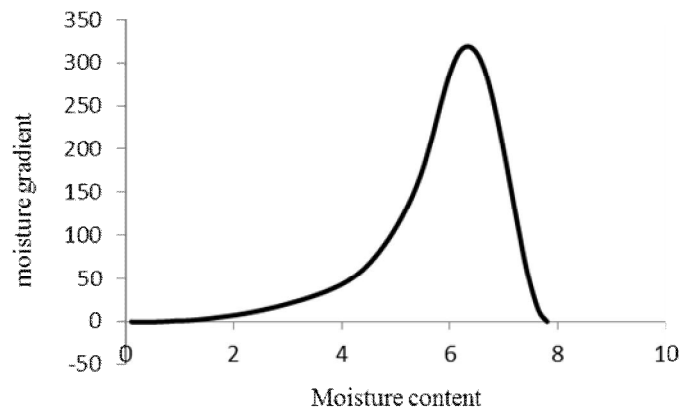

Fig.5.relation between simulated maximum moisture gradient and moisture content

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